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THESIS

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by

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Lieutenant, United States Navy

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Submitted in partial fulfillment of
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MASTER OF SCIENCE
IN
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ABSTRACT

Transmission lines suitable for use in the millimeter and submillimeter length bands are described and compared. The advantages of using optical transmission techniques to as low a frequency as possible are discussed. Certain advantages are seen to exist in the use of oversize rectangular guide instead of circular electric mode propagation. Various suggestions for dielectric transmission lines are investigated, the capabilities of each discussed.

The writer wishes to express his appreciation to Marvin Waldman of Sylvania Reconnaissance Systems Laboratory for suggesting the study, and to Professor Donald Stentz of the U. S. Naval Postgraduate School for his suggestions and encouragement.

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PART 1

INTRODUCTION

1.0. Introduction to Ultramicrowave Region

Increased use of presently explored frequency space is forcing the development of new techniques and the exploration of additional bands to handle the expected volume of traffic of the near future. One interesting part of this expansion is the investigation of the millimeter and submillimeter wavelength bands, sometimes referred to collectively as the ultramicrowave (UMW) region. Together these bands represent 99 times the width of the spectrum from D. C. to 30 Gc/s. In addition to the great width of the band, the UMW region has other interesting properties; first, the region represents two decades of relatively unexploited space in the center of the otherwise highly utlized characteristics of great interest in the fields of selective and secure communications. The extreme width of the band is in itself a measure of security hindering the interception of a signal. In addition, very directive antennas can be of compact size; at 200 Gc/s a one foot parabolic reflector has a beamwidth of $\frac{1}{2}^{\circ}$ and a gain of nearly 40 db. Such an antenna would be entirely practical as part of a line of sight communications system having an intercept probability lower than infrared systems, and capable of an information rate almost beyond comprehension. A six percent bandwidth would result in a channel 12 Gc/s wide.

A complete area of interest is that of airborne systems. In addition to the small size of the radiating elements, advantage could be taken of the atmospheric effects on the region to achieve results

unobtainable at lower frequencies. Examples of such devices might include air-to-air communications or air search radar virtually undetectable from the ground, or high resolution radar of limited horizontal range on the surface for navigation or ground mapping purposes.

Any millimeter wavelength system must be designed to make use of the effects of the atmosphere on propagation. These effects include frequency selective absorption by oxygen and water vapor, and scattering by water drops. The latter is, of course, variable and dependent on the size of the drop with respect to a wavelength. Consider that at the wavelength of visible light fog scatters in the same manner.

One other factor is currently of importance to the component designer. Present research in millimeter power sources has not presented a means of generating large amounts of power; a major breakthrough is seen necessary to permit the generation of high power coherent signals at UMW frequencies. Greater output is available from noncoherent sources, at no better efficiency than the coherent sources, but present modulation capabilities with these sources is comparatively low.

The preceding paragraphs show some of the reasons for interest in the UMW region and imply the requirements for equipment design. Thus the requirements for a transmission line at UMW can be established. It must have low attenuation in order to add as little loss as possible to an already weak signal, wide bandwidth for size standardization or to match to tuning range of wideband devices such as intercept receiving equipment, and low susceptibility to spurious signals. At microwave frequencies the dominant mode rectangular waveguide performs all these functions in an acceptable manner, but its high attenuation forces us to look at other

lines in the UHF region.

This paper will attempt such a survey. In particular, it will be concerned with the search for a line or lines meeting the above requirements as well as additional requirements imposed by use in and around equipment. It will be shown that certain waveguides have admirable qualities for long distance communication, but these same guides are not necessarily the best choice for the equipment designer who must deliver a signal only a few feet, but not in a straight line.

The remainder of this part will present the problem and outline the basis of comparison to be used on the proposed lines. Part 2 is a more precise description of each of the lines considered, while part 3 will be an attempt to evaluate them in a manner helpful to one who must select one or a combination of these lines for use in equipment.

1.1. Summary of Ultramicrowave Region History.

Radiation at wavelengths below one centimeter was produced and detected before 1900[3], particularly by Bose who, in 1896, achieved five millimeter operation using a spark gap source. In 1923 the entire range from infrared to microwave was studied by Nichols and Tear[4] using Hertzian dipole technique in quartz mercury lamps. In spite of these early studies, the present limits of production equipments are about 40 Gc/s, with experimental equipments[5,56] extending to 150 Gc/s. Unclassified commercially available sources are all restricted to use below 90 Gc/s[9], with laboratory experiment types having been operated to 511 Gc/s[10-13], but producing very little power output from large and complicated units. Theorotical studies made on the froquency limitations of sources[13-16] have forecast the ultimate limits on presently used microwave structures between one and three millimeters. Some extention is available by frequency conversion and multiplication, but entirely new techniques are indicated for use below about five millimeters for power sources, and one millimetor for local oscillators.[10,17]

Crystal detectors have been developed which promise useful operation to about 100 Gc/s[18-20] with possible extention through refined techniques to 1000 Gc/s. Beyond that range, present experimentation (now carried on at 100-500 Gc/s) is with optical[21] and quasi-optical[22] (infrared) techniques. The infrared methods now in use are characterized by extremely slow response, the fastest of them giving an upper frequency limit of about one megacycle, and the more sensitive detectors usable only to about 10 c/s.

An important consideration in the UMW band is the effect of oxygen

and water vapor absorption[1,]. The effect varies with frequency, altitude, and humidity, but has been extensively investigated so it can be considered predictable. In addition to the absorption, scattering by raindrops has been experienced. This is to be expected, since the drops are in the order of magnitude of a wavelength at UMW. A knowledge of environmental conditions is thus essential to the equipment designer in order that a proper choice of operating frequency may be made. As examples, an air-to-air communication system would be best situated near the peak of one of the absorption bands, but a ground mapping system, or any surface to surface equipment would find the same frequency highly undesirable, preferring instead one of the troughs in the curve.

The attenuation constants quoted for hollow waveguides assume that some action has been taken to assure the guide is dry and oxygen-free; pressurizing with nitrogen or one of the inert gases is recommended. It is realized that such an assumption will have little effect on the total attenuation in the short runs normally found around equipment, but the extra attenuation would be noticable on any appreciable length of guide, and would vary with diurnal weather changes. The basic technique is nothing new, since most airborne systems are presently pressurized, but with air, and for the purpose of increasing the breakdown potential of the guide. The UMW pressurization would be more gainfully employed in surface installations than in airborne equipment because of the greater partial pressures of oxygen and water vapor near the earth's surface.

1.2. Ultramicrowave Transmission Line Requirements

The variety of proposed systems for UMW use make the selection of the proper transmission line somewhat less trivial than at lower frequencies where one assumes the use of H_{10} and can compute expected system performance using one of the standard works[(24-26)], using standard guide sizes. It seems proper that we consider the requirements of a generalized system before discussing the possible means of realization at UMW. By that means it is possible to evaluate each line against a standard made up of conditions typical of a practical equipment. Proper weighting of the separate evaluations should then indicate the most desirable line for a particular installation.

Microwave Theory and Techniques[(42)] suggests a general approach in its breakdown of transmission line losses into (a) coupling loss (b) attenuation loss (c) reflection loss, and (d) mode conversion loss. In addition to these losses, (e) radiation loss is of interest to the equipment designer since radiation is generally bilateral and the radiating line is at once a source and receiver of spurious signals. In the discussion to follow, reflection loss can generally be assumed small except in specific cases such as mode converters and filters. Coupling loss will be about the same for all hollow guide types, and will be mentioned whenever it differs significantly from an equivalent rectangular waveguide situation. Attenuation and mode conversion losses will therefore be the primary quantitative basis of differentiation between the types to be considered. The self shielding properties of each line will usually be evident, but will be discussed in those cases having finite leakage past the bounds of the guide.

Within the frame of the points above, the following points are of interest and will be covered in each case by an investigation of the governing conditions, and such quantitative data as seem necessary or desirable.

(1) Attenuation factor of a straight section of line, nominally one meter long, with attention paid to the effect of manufacturing/assembling imperfections, and losses incurred by any necessary filters.

(2) Attenuation in a 90° bend; in rectangular guide the bend radius is normally 3/2 the guide dimension in the plane of the bend as measured to the guide centerline. If such a radius cannot be attained, reasons why. Also included if applicable are the effects of other angles than 90°.

(3) Necessary and available accessories in the use of the line; size and loss penalties of these parts.

(4) Realizable bandwidth of a system made up of the components above.

1.3. Proposed Transmission Lines

In addition to the $H_{10}^{\square}$ rectangular guide used as a standard of comparison, the lines under consideration resolve themselves into several headings with variation in detail being commonplace in each group.

The first such grouping is the H_{01}^{o} mode in a hollow circular pipe at frequencies considerably above cutoff. The monotonically decreasing attenuation of this mode has been known for some time, as has the practical difficulty of utilizing a mode which is not the dominant mode in the guide at a frequency sufficiently high that many additional modes can propagate [24,27,28]. In addition, the H_{01}^{o} is not a normal mode of

curved circular pipe under any conditions. Considerable study has produced reasonable results in bending this mode, but only with high loss or narrow bandwidth. Several variations on the solid wall circular guide for this mode include spaced disc[47] and helix[44] guides.

Another classic scheme is that of the surface wave transmission line[46]. In practical form this is a single wire having a corrugated surface or a dielectric coating which has been used as low frequencies (60-200 Mc/s and has been proposed as useful at UMW.

Those lines characterized by one or more plates of conducting material in contact with dielectric include the very interesting H guide [31] which is purported to share with the H^o_{0n} modes a decreasing attenuation with increasing frequency. Others in this group are the various image lines[32] whose properties more closely resemble the dielectric rod and tape lines from which they were derived.

Although somewhat startling at first meeting, it becomes a natural continuation of electromagnetic theory that at higher frequencies dielectrics can guide electromagnetic energy[45]. Common examples presently in use include the leaky rod antennas used at microwave frequencies, and the use of Lucite to guide beams of visible light. Since the dielectric approach represents a return to the transmission of energy principally within the body of the guiding substance, it is to be expected that somewhat high attenuation will be found in the limiting case. An attempt to reduce this attenuation in the middle of the operating range these lines have been split and mounted on a conducting image plane as mentioned above, and special shapes are

being investigated which may be able to reduce the radiation of the line at frequencies below that at which dielectric losses become predominant [33].

2.0 SYSTEM CHARACTERISTICS

2.1. Rectangular guide in H_{10} dominant mode operation

The characteristics in this section may of course be found in, or derived from the information available in handbook form[25,26], but are presented here as a convenient standard against which to compare the lines to be considered below.

Fig. 2 illustrates the coordinate nomenclature of the line.

Straight line attenuation in this mode is published for standard sizes of guide, or may be calculated from the equation;

$$\alpha = \frac{R_s}{\eta_f b}\left[\frac{1+\frac{2b}{a}\left(\frac{\lambda}{2a}\right)^2}{\sqrt{1-\left(\frac{\lambda}{2a}\right)^2}}\right] \frac{\text{nep.}}{\text{met.}} \qquad (2.1\text{-}1)$$

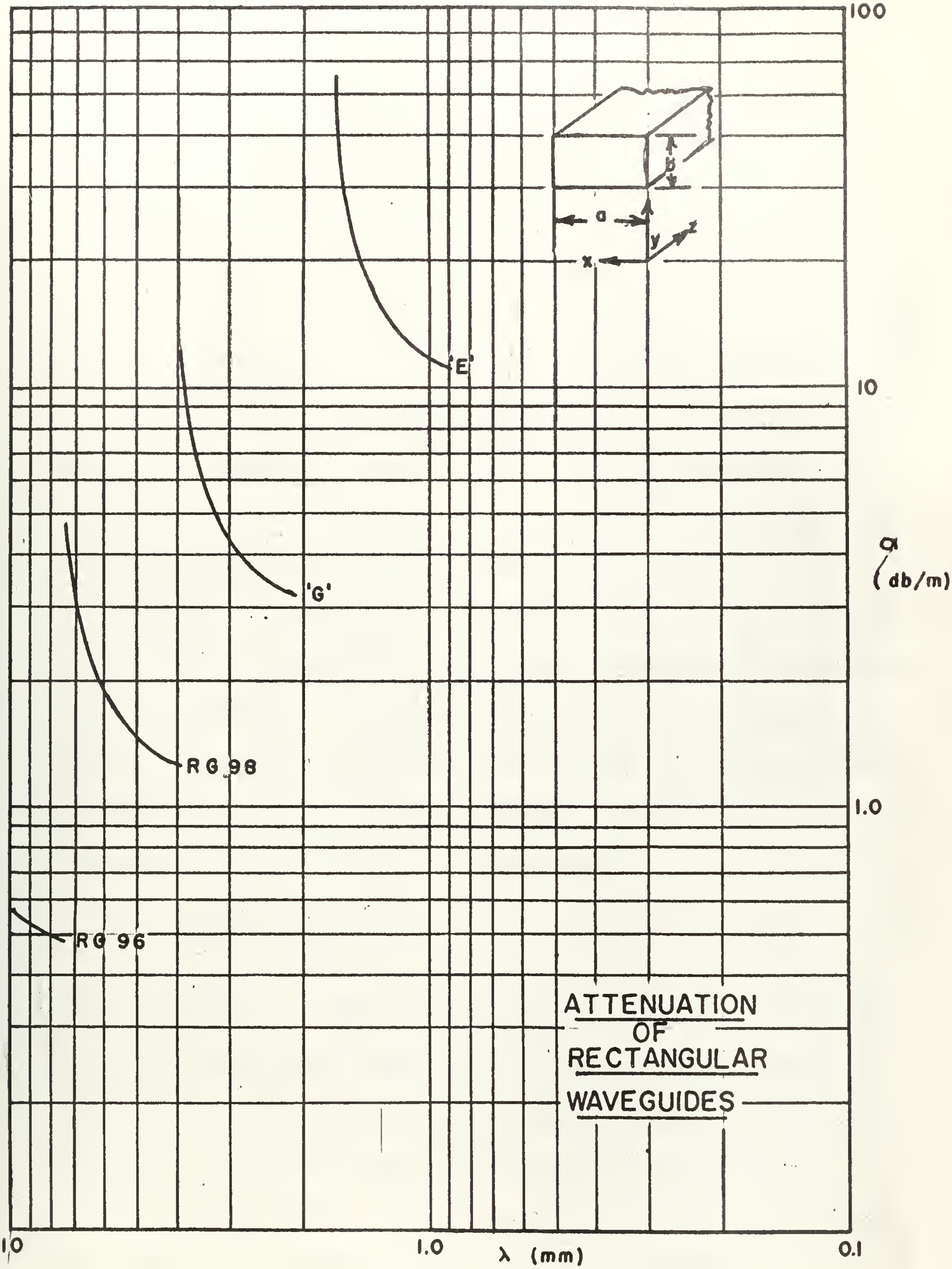
More easily evaluated equations for specified conditions such as copper guide and air filling are available, but are not so well suited to the present requirements. A summary of standard guides in the UMW region is shown in table 2-1 below.

Rectangular waveguide bends have presented little problem, and are available commercially. The average effect on the line is a maximum VSWR of 1.05 over the published frequency range of the guide in use. It may be seen from the field equations for this mode that regular bends (ie, circular, without changing appreciably the cross-section of the guide) in either E or H plane cause little additional loss, and since the guide is operated in the dominant mode, no effective mode conversion occurs.

guide	in. dim. inches	f range Gc/s	db/met. atten const.	Remarks
RG 52	.90"x.40"	8.2-12.4	.211-.144	X band, copper
RG 96/U	.28 x.140	26.5-40.	.718-.492	silver lined
RG 97	.224x.112	33.--50.	1.016-.686	"
RG 98	.148x.074	50.--75.	1.732-1.281	"
RG 99	.122x.061	60.--90.	3.06-1.71	"
"G"	.080x.040	90.-140.	5.0--3.25	"
"F"	.051x.0255	140.-220.	10.1--6.3	"
"E"	.034x.017	220.-325.	16.8-11.4	"

Table 2-1. Characteristics of Rectangular Waveguide at Ultramicrowave Frequencies.

Present waveguide techniques are of course based primarily on the use of rectangular guide; as a result, there is no lack of available fittings for any of the standard sizes of this guide. As can be seen from the table above, the bandwidth of the guide is adequate for our requirements; the most frequency sensitive elements of an installation are usually found in matching devices such as TR and ATR switches, and in cavities. Tunable crystal mixers have been produced over about half the UMW range, but wideband mixer designs are rare. Upper frequency limit is a function of the tolerable ammount of attenuation in a given application, and the ability of manufacturers to adhere to the increasingly difficult dimensional requirements. It must be admitted that "E" guide is for laboratory use only, and that the attenuation of RG-99/U is considerably higher than one is usually willing to accept, even for the convenience of using more nearly standard components than those to be described in the following sections. A means of extending the usable frequency range of rectangular guide will be discussed in 2.7.



100
10
1.0
α (db/m)
'E'
'G'
RG 98
RG 96
a
b
x
y
z
ATTENUATION OF RECTANGULAR WAVEGUIDES
10
1.0
λ (mm)
0.1

Figure 1

2.2. Circular Waveguide in H^{o}_{01} mode.

Circular hollow waveguide operated in the circular electric or H_{01} mode is almost unique among transmission lines in the respect that its attenuation constant is a monotonically decreasing function of frequency. Thus, it is theoretically possible to attain any desired level of attenuation by proper choice of frequency and guide diameter. This characteristic was not sufficiently important at lower frequencies to overcome objections to the larger guide dimensions compared with rectangular guide, or the practical problems associated with the use of the circular electric mode. At UMW frequencies, however the ability to use a guide of almost arbitrary size is an advantage which has spurred considerable research in the past few years.

The H_{01} is not the dominant mode in a circular guide, but the fourth or fifth to appear in order of cutoff frequency. Two associated problems are present; the E_{11} mode has the same cutoff frequency as the lowest order circular electric mode, and within a band equivalent to the usable band of the dominant rectangular band, 14 modes can propagate. In an arbitrary guide, approximately

$$M = 2.55 \left(\frac{D}{\lambda}\right)^2 \quad ; M > 10 \qquad (2.2\text{-}1)$$

modes can propagate. Recent studies have been directed toward making the H_{01} mode effectively dominant [28,48], and reducing the number of modes actually able to propagate in a given pipe diameter [44]. Others have attempted to attain measured attenuations approaching the theoretical values [38]. In order to better understand the progress being made, a better description of the mode is in order.

The lowest order circular electric mode has field components:

$$H_z = H_1 J_0 \left(\frac{2\pi r}{\lambda}\right)$$

$$H_r = -j \frac{\lambda H_1}{2\left(\frac{\lambda}{\lambda_c}\right)} J_0' \left(\frac{2 r\pi}{\lambda_c}\right) \qquad (2.2\text{-}2)$$

$$E_\theta = j \frac{H_1 \lambda}{\pi 2 \eta_f \left(\frac{\lambda}{\lambda_c}\right)^2} J_0' \left(\frac{2 r\pi}{\lambda_c}\right) \quad ; \quad \lambda_c = 1.640a$$

and an attenuation constant:

$$\alpha = \frac{R_S}{\eta_f a} \frac{\left(\lambda/\lambda_c\right)^2}{\sqrt{1 - \left(\frac{\lambda}{\lambda_c}\right)^2}} \qquad (2.2\text{-}3)$$

The implications of (2.2-3) are obvious as pointed out in the introductory paragraph. This is, however, only part of the picture. Because of the operating conditions many other modes can propagate in the guide, and any deviation of the guide from a straight circular cylinder can cause mode conversion loss by coupling the H_{01} mode to one or more other modes. Appendix A will expand on the coupling factors as found by theory and experimentally verified, (37,38) but the result is that between any two modes there can exist a bilateral coupling constant, c_{mn}, which is excited in proportion to a nonuniformity of the guide, B, to form the coupling coefficient, C_k. For regular or finite discontinuities, a summation is then made; for small, irregular, or numerous perturbations, the additional loss has an average value very close to the RMS coupling factor taken over the length of the guide, or,

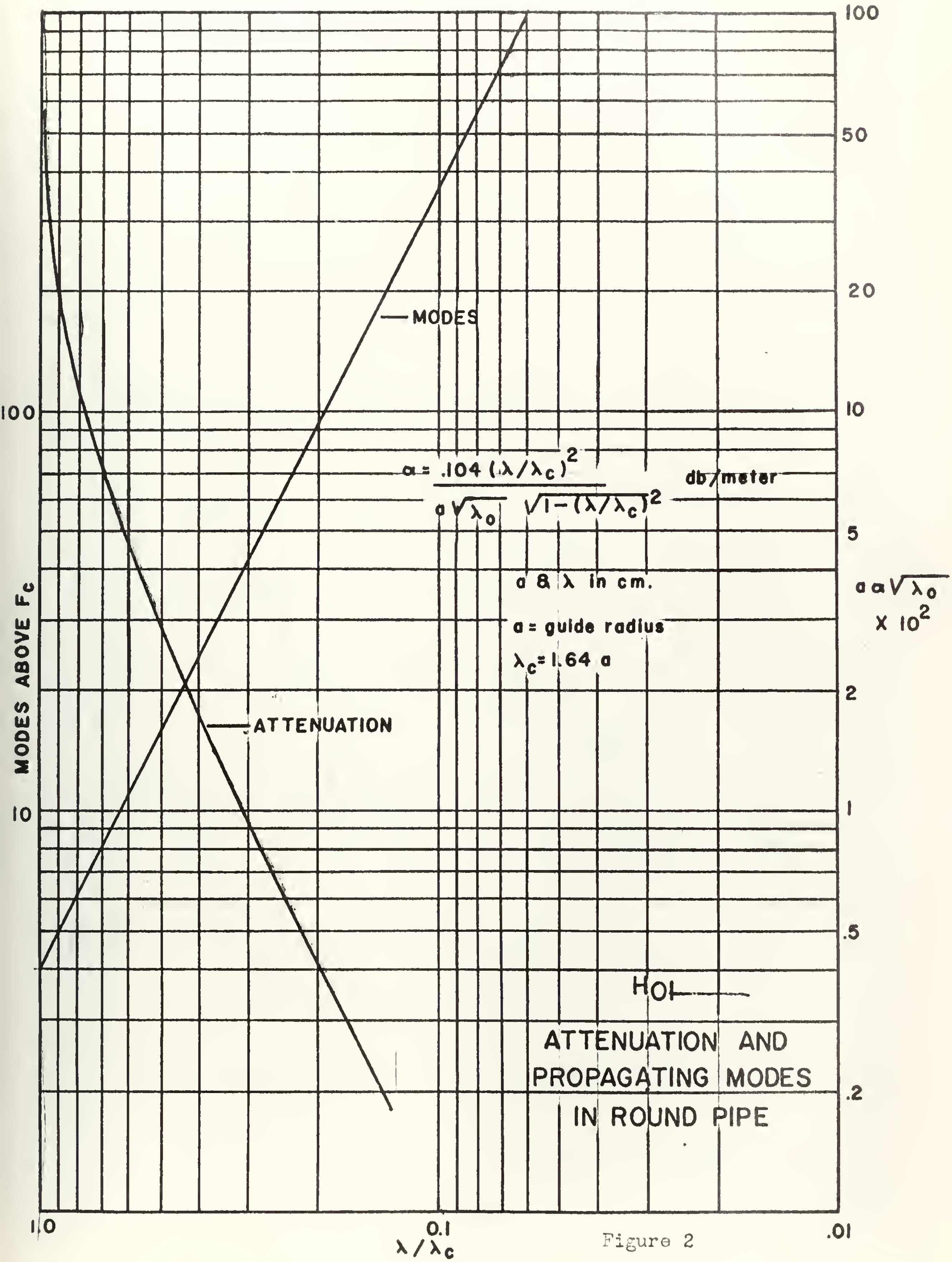

MODES
ATTENUATION
a = .104 (λ/λc)² / a√λo √1−(λ/λc)² db/meter
a & λ in cm.
a = guide radius
λc = 1.64 a
H01
ATTENUATION AND PROPAGATING MODES IN ROUND PIPE
MODES ABOVE Fc
aα√λo X 10²
λ / λc
1.0
0.1
.01
100
10
100
50
20
10
5
2
1
.5
.2

Figure 2

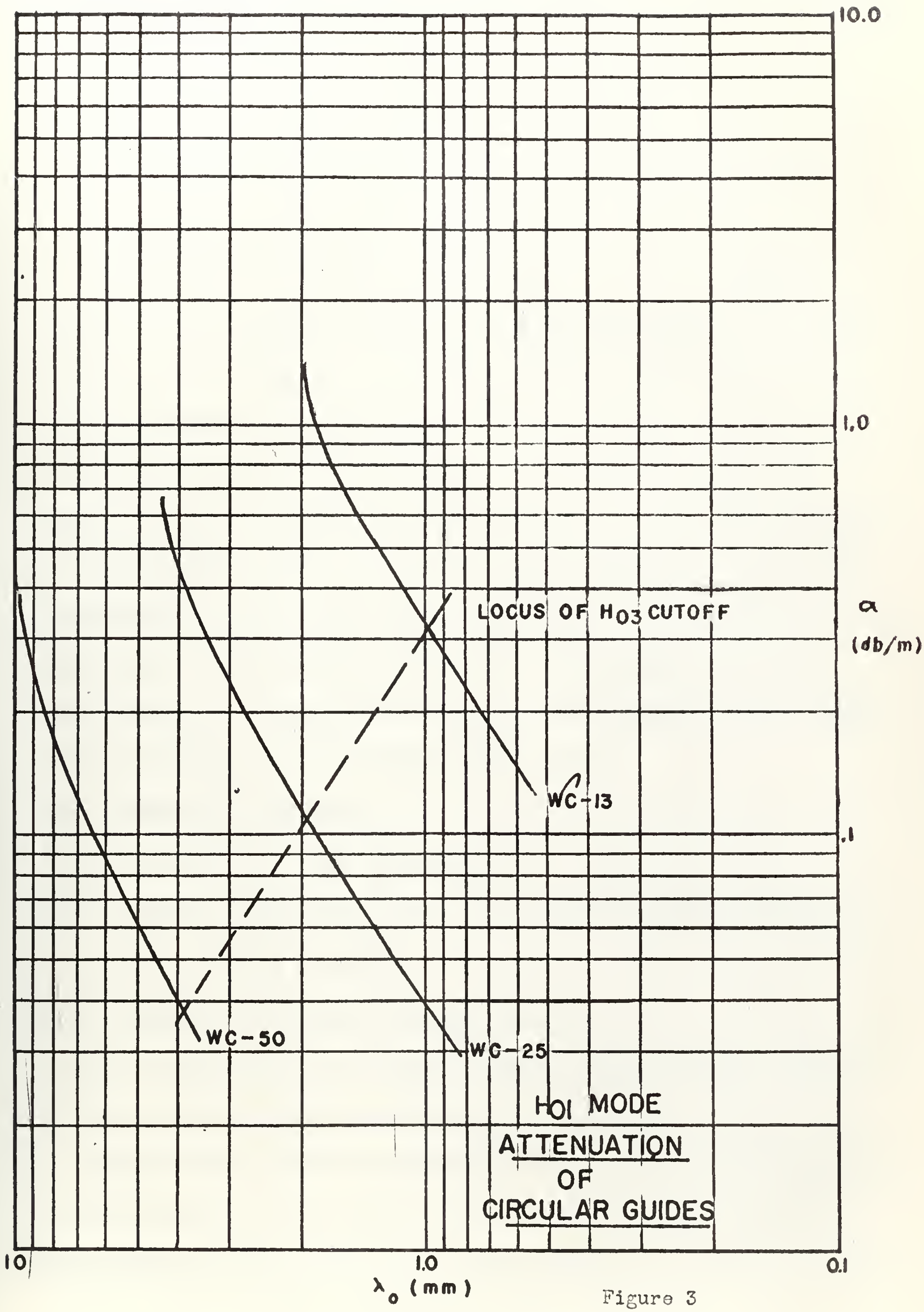

10.0
1.0
.1
α
(db/m)
LOCUS OF H_{03} CUTOFF
WC-13
WC-25
WC-50
H_{01} MODE
ATTENUATION
OF
CIRCULAR GUIDES
10
1.0
0.1
λ_o (mm)

Figure 3

$$\langle \Delta\alpha \rangle = \left\langle \frac{|C_k|^2}{2} \right\rangle = \langle \Lambda \rangle \quad nep. \tag{2.2-4}$$

Since the coupling is random and bilateral, there occurs a deviation

$$\langle \delta\Lambda \rangle = \frac{1}{2} \langle \Lambda \rangle^2 \quad nep. \tag{2.2-5}$$

about the mean expressed in (2.2-4). This deviation has an effect on signal output similar to a high VSWR on the line, but with a periodicity determined by the difference in phase velocities of the modes concerned rather than the wavelength. Both these expressions are additive to the attenuation in (2.2-3) and must be considered for each propagating mode capable of coupling to the desired mode. To a first approximation, valid in well made straight guide, the process is calculable on a superposition basis, and is predictable from cold measurements. The possible variations from straightness and the modes coupled to H^o_{01} by them are summarized as follows:

(a) diameter variation; H_{on}

(b) ellipticity; H_{2n}, H_{21} principally

(c) deviation from straight line, no net change in direction; H_{1n}, principally H_{12} and H_{11}

(d) bending, E_{11} followed by effects of (c)

Coupling coefficients for the more important modes have been computed by Morgan and are listed in Appendix A.

It is at this point that the helix wound guides can be appreciated. All modes except those of the circular electric family have axial components of current in the wall; these modes can be highly attenuated by use of an anisotropic wall which conducts only transversely.

The most practical approximation so far developed is the guide made of close-spaced fine wire wrapped with as small a pitch as possible, and backed with a lossy dielectric. This guide is sufficiently lossy to modes other than the H_{on} family that the periodic losses shown in (2.2-5) are not significant, the total attenuation being the theoretical mode loss plus the RMS conversion losses introduced by manufacturing and laying imperfections. By varying the wire size and pitch angle [39] other modes can be selectively propagated, making the helix concept a useful design tool.

In addition to direct mode conversion loss, there exists a distortion limitation caused by the different propagational velocities of the different modes. [40] If energy coupled to a spurious mode is allowed to propagate a finite distance before reconverting to H_{01} the reconverted signal will lag that part which has been propagated entirely in H_{01}. The extent to which this distortion can be tolerated will depend on the particular design.

guide	i.d.	diameter tolerance	ellipticity tolerance	freq.
WC 50	.500"	.0005"	.0005	34-48Gc
WC 25	.250	.00025	.0001	69-96
WC 13	.125	.00025	.00005	139-192

Table 2-2. Typical Standard Circular Guide.

Satisfactory operation in straight circular guide is less difficult than any presently known method of conducting the circular electric wave around bends. The mode is not a normal one in curved circular guide as is shown in Appenddix A, but is degenerate with E_{11}; the resulting energy transfer is:

$$\text{Amp. } H_{01} = \left| \cos\left(\frac{\pi}{2}\frac{\theta}{\theta_c}\right)\right|$$

$$\text{Amp. } E_{11} = \left| \sin\left(\frac{\pi}{2}\frac{\theta}{\theta_c}\right)\right| \tag{2.2-6}$$

where

$$\theta_c = \frac{\pi}{2.32}\left(\frac{\lambda}{a}\right)\frac{1}{\sqrt{\epsilon_r}}$$

is the critical angle for complete conversion. θ_c is seen to be frequency dependent, but independent of bend radius.

In addition to the H_{01} - E_{11} coupling, those modes which coupled to H_{01} in straight guide are still present. As shown in the previous section, H_{12} is coupled to H_{01} by deviations from straightness; the coupling coefficient of these two modes is even higher than that between the H_{01} and E_{11}. They are not so strongly coupled in presently considered bending because of the different nature of the coupling, but when the E_{11} mode is so compensated that its coupling is negligible, the bending radius is still limited by the acceptable loss to H_{12}.(37) Other modes also couple as discussed in the previous section.

Among the devices which have been considered to reduce bending loss in the H_{01} mode are the use of dielectric loading by strips(37) or discs(41), corrugation of the inner surface and the use of continuous or close spaced mode converters filters of more common metallic design.(24)

The object of these metallic filters is the reduction of conversion to spurious modes, or the reconversion of these other modes back to H_{01} as soon as possible. The corrections are usually frequency sensitive since all modes except the selected one for transmission are

reflected to a degree which depends strongly on wavelength.

At present, circular waveguide is used primarily as an accessory to rectangular guide for long straight low loss runs. Experimental work at higher (UMW) frequencies has caused the evolution of special components in a limited number of sizes. These items include helix guide filter sections, rectangular to circular mode converters, and a bend section (in 3/16" guide) having a bend radius of 4.5". This section is made of corrugated straight tubing which is subsequently bent to the required angle. A 90° bend made in this manner has an insertion loss of .2 db over a 6% bandwidth in the 75 Gc/s region. Bell Labs [20] has carried out preliminary design work on a slow wave structure to directly excite the H_{01} mode; more work in this direction is indicated, as is the development of a mixer to detect H_{01} directly, and operable over a band approaching waveguide bandwidth. Insertion feeds and hybrid directional couplers have been described in the literature [55,53] but in each instance, they use rectangular guide as the coupling out structure.

Realizable bandwidth of an all circular waveguide system would be limited by the degree and type of mode suppression used, and by the degree of compensation accomplished in bends. Since these items also effect attenuation at the center frequency, the specific choice would be a result of design parameters. It should be possible to achieve 20-40% bandwidth with losses of 1 db/meter including one 90° bend using present techniques. Extended use of this guide and the wide interest in it by long distance communications and radar designers should lead to even lower losses through the development of new techniques of bending and mode selection.

2.3. Dielectric Rod and Tape Guides

It has been recognized for some time that dielectric materials having a relative permittivity greater than that of the surrounding medium can exhibit a guiding influence on electromagnetic waves. Two examples in present use are leaky rod dielectric antennas, and lucite guides for visible light. At UMW frequencies it is practical to guide the radio frequency energy by such means. Several variations has been proposed, including rod, tube, tape with conducting image plane, and so called H-guide. Academic discussions of dielectric propagation can be found in Southworth[24] and Ramo and Whinnery.[60] Since the basic rod exhibits the properties of the group, it will be investigated first, then those results applied to a consideration of the modified guides.

The guiding action of a dielectric depends on the difference in phase velocity across the interface. Below the so-called divergence (cutoff) frequency at which $\beta = \omega\sqrt{\mu_0 \epsilon_0}$ there can be no guiding action. Above that frequency guidance becomes more effective as frequency increases until in the upper limit, $\gamma = j\beta = j\omega\sqrt{\mu_0 \epsilon_1}$ for a lossless dielectric. At that time there is no field outside the guide.

In determination of attenuation, three forms of loss may be considered. That part of the energy inside the guide suffers dielectric loss due to imperfect dielectric; the energy radiated or that carried too far from the guide to be economically recovered is considered radiation loss; and in some cases the conduction loss through the interface must be considered, though this is usually modified by the launching devices used.

A study of modes present in the guide [58] shows that most

propagating modes must have axial components of both E and H; E or H modes can be propagated only if their fields have axial symmetry. Three modes are of importance; the dominant HE_{11}, H_{01} and E_{01}. The first is unique in having a divergence frequency of zero. The lowest order circular modes are degenerate, having a divergence frequency at $d = .626\,\lambda$. In all three cases, loss is due to the factors mentioned above; the dielectric loss is proportional to the power carried in the guide, thus to frequency. At $d/\lambda = 1$ all three modes have $0.9 \leq \frac{\alpha}{\alpha_d} \leq 1.0$ where α_d is the infinite medium loss,

$$\alpha_d = \frac{\sqrt{\epsilon'}\,\pi \tan\delta}{\lambda} \qquad (2.3\text{-}1)$$

Due to its lack of a divergence frequency, HE_{11} can be used over a wider band than the other two modes and has the further advantage of compatibility with H^{o}_{11} and $H^{\square}_{10}$. The mode is also the most easily polarized of the three, simply by use of oval or elliptical dielectric guide.

Presently available loss tangents are seldom better than 10^{-4}, requiring for reasonable attenuation that only a small part of the energy be allowed to travel within the guide. Thus (2.3-1) must be corrected to account for the reduced extent of the dielectric. There are three means for finding the approximate attenuation, though they are all derived from the same source, an evaluation of the fields in and around the guide. Thus, one may evaluate the Poynting vector, make use of available charts [20], or make direct computations, using the following relationship derived from the field evaluation;

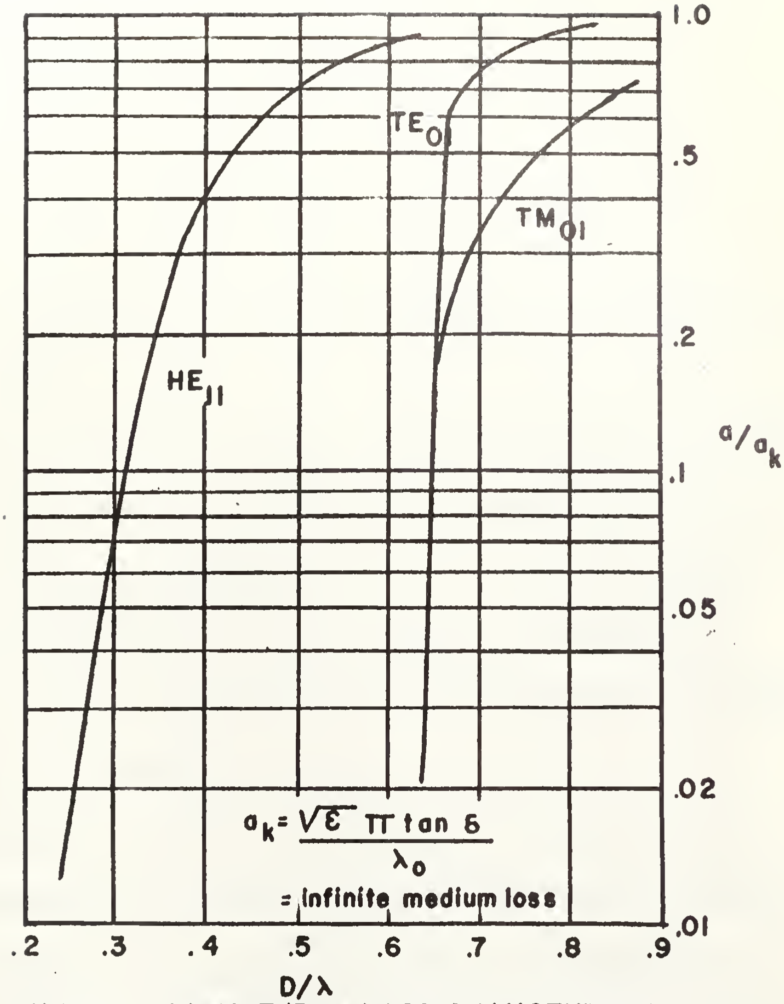
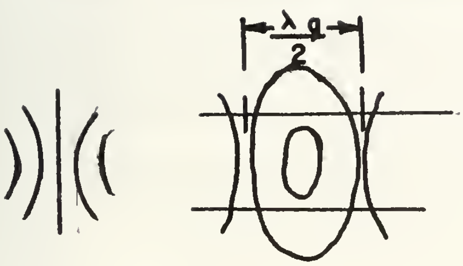
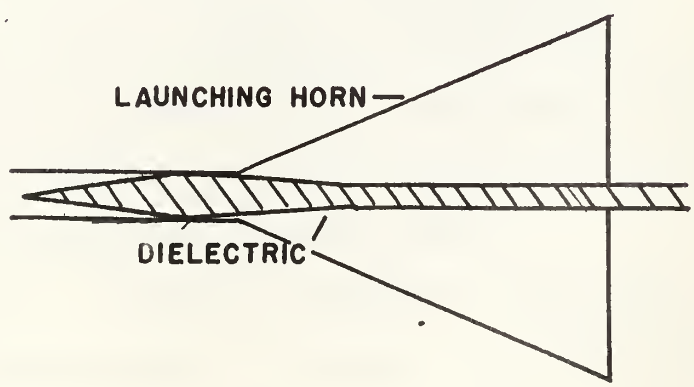


FIG. 4a. DIAMETER vs LOSS, POLYSTYRENE



4b. E FIELD OF HE_{11}.



4c. LAUNCHING HE_{11} MODE.

Figure 4

$$\alpha = 27.3 \left(\frac{\epsilon \tan \delta}{\lambda} \right) R \qquad (2.3\text{-}2)$$

where

R= complex function of operating conditions, see appendix of Ref. 59, and ref. 64.

Attenuation can be reduced by use of special guide shapes which tend to reduce the dielectric attenuation while maintaining a given amount of guidance and an acceptably low level of radiation loss. Among these are hollow tubing[62, 33] or cruciform extrusion[33]. Since these guides are still all dielectric, the bandwidth and ultimate attenuation are not much affected.

Bending in dielectric guides is not quite the problem that it is in circular electric mode metal pipe, primarily because one mode only is allowed to propagate, but it is generally lossy. Losses are predictable and vary with the polarization of HE_{11} used, the degree of guiding action maintained in the bend, and the radiation loss experienced. The first can be controlled by use of non-circular guide shape, while the latter two can be aided by the use of larger guide and/or the use of reflecting plates so placed as to reflect energy radiated from one side of the bend back on to the line leaving the bend. In this respect note that if $\theta > \theta_c$ radiation can occur only in a bend through distortion of the normally purely reactive field.

Another possible means of eliminating radiation loss from bend sections is the use of reflecting plates so arranged as to reflect the radiated signal back onto the line in the region immediately after the bend. The metal, of course, affects the attenuation by I^2R loss, but in many cases this is much less than either the radiation loss, or the

dielectric loss in a completely dielectric system.

Several components have been built or adapted for use with dielectric lines, among them being launchers, couplers and antennas. The bandwidth of the low attenuation region is relatively small, but Bell Laboratories in particular[20] has found these lines useful as patch cords at millimeter wavelengths. Horn launching from rectangular or dominant mode circular guide has proved most efficient in exciting the lines. The remaining problems are the required small diameters of guide with presently available loss tangents, the attendant necessity of frequent supports for the guide, and the necessity for shielding to reduce spurious signal pickup; the latter has been solved in theory by use of lossy dielectric shielding to absorb the necessary power without creating hollow waveguide type modes as would occur in metal guide. The practical limitation to all types of shielding however, is the ability to properly align the dielectric within the shield, and maintain that alignment in varying atmospheric conditions.

2.4. Dielectric Image Line

Several variations on dielectric surface wave lines may become of extreme interest in the future; two of these lines showing promise are the image lines and the H-guide. Physically, the two bear some similarity to each other, but the operation is somewhat different, so they will be treated separately.

Dielectric image line[32,62,64] is essentially dielectric rod guide operating in the HE_{11} mode, but split along the axis of symmetry, that "diameter" then being mounted against a conducting image plane sufficiently wide that fringing is negligible. The image plane adds conductor loss to the dielectric loss of (2.3-1)

$$\alpha_m = 69.5 \left(\frac{R_s R'}{n \lambda} \right) \text{ db/meter}$$

$$R' = f(c); \text{ see ref. } 61 \qquad (2.3\text{-}2)$$

but affords the necessary support for the dielectric, allowing smaller sections of weaker and generally less lossy material to be used. Although only semicircular guide has been investigated theoretically, Wiltse[64] has found superior bandwidth and lower attenuation from the experimental use of thin tapes as dielectrics. He has also constructed detectors to operate directly in the tape guide. Other components for use in this line are described by King,[65] while King and Schlessinger[59] have reported lower frequency experimentation with various image line obstacles. It goes almost without saying that low loss bending is possible in the guide in the plane perpendicular to the image plane, and fairly gentle twists to properly align this plane should cause no difficulty.

King's experiments[65] point out that low losses are afforded by concave E plane bends, that considerably more loss is incurred (5 db approx) in H plane bends, and that convex E plane bends are more efficient radiators than transmission lines unless considerable increase in the dielectric crosssection in the bend.

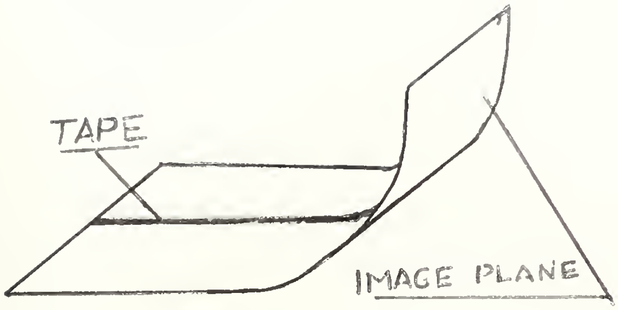


Fig. 5a

Concave E Bend

Fig. 5b

HE_{11} mode Electric Field Pattern

2.5. H-Guide

H-guide is a specially configured guide based in theory on the characteristics of rectangular waveguide, but having operating characteristics strongly influenced by the dielectric slab contained in the line.

As the sketch shows (fig.6), H-guide consists of two parallel conducting planes separated by a dielectric slab. Normal operation is in the HE_{11} mode, fed by rectangular guide. It is argued by H-guide proponents(31) that this guide has a monotonically decreasing attenuation similar to that of the circular electric mode in circular pipe, but this is true only for the metal loss, thus for guide having a totally lossless dielectric. Studies(65,66) of the guide using loss tangent of 10^{-4} have shown the total attenuation of the unit experiences a minimum dependent on the dimensions of the dielectric and its dielectric characteristics but in general is somewhat better

than dominant mode rectangular guide of similar dimensions. Additional advantages might be realized by laminating the dielectric in a plane either parallel to or perpendicular to the metal plates,[31] so that maximum field strength occurred in an enclosed air space. Proper engineering should make this a flexible and useful transmission medium in the range of frequencies around 100 Gc/s, Among its advantages may be the ease of manufacture, by a combination of extrusion of the dielectric and electroplating the required metal on the outer surfaces. Several components have been proposed using this method of construction,[65] including hybrid rings and antennas.

Another proposed mode of operation is the H_{mo} family but the required close plate spacing and resulting high attenuation (about equal to rectangular guide) make it of more interest at lower frequencies where its chief advantage is the ability to handle more power than comparable rectangular guide. An advantage of H-guide at present is its compatibility with rectangular waveguide, operated in H_{10} mode; excitation can be acomplished with little loss by a simple taper transition.

Bending, too, is more easily accomplished in this guide than in other dielectric guides, especially E plane bends which are self-shielding; additional loss in primarily increased metal loss and small in smooth bends. H plane bends are not as lossy as plain dielectric rod, but are best avoided because of large radiation loss unless reflecting plates are used.

From the following formulas for H guide it can be seen that metal losses follow a -3/2 slopo (sometimes published as the attenuation curve of H guide) and are negligible over most of the operating range in comparison with the dielectric attenuation at any presently realizable

value of tan δ. An experimental section of this line constructed to mate with RG-96/U has been evaluated by Griemsmann and Birenbaum[62,63] in conjunction with a UMW study effort at Brooklyn Polytechnic Institute[56]. Some characteristics of this line are shown in fig. 6.

H-Guide Equations

a. Metallic attenuation

$$\alpha_m = \frac{2R_s}{b\eta_o}\left(\frac{\lambda_o}{2b}\right)^2\left(\frac{\lambda_g}{\lambda_o}\right)\frac{\epsilon}{\tau}\cdot\frac{1+\dfrac{2\tau}{k_o a\sqrt{\tau-1}\,(\tau\epsilon+\tau-\epsilon)}}{1+\dfrac{2\epsilon}{k_o a\sqrt{\tau-1}\,(\tau\epsilon+\tau-\epsilon)}}$$

b. Dielectric attenuation

$$\alpha_d = \frac{\pi\epsilon\tan\delta}{\lambda_o}\left(\frac{\lambda_g}{\lambda_o}\right)\cdot\frac{1+\dfrac{2(\tau-1)(2\tau-\epsilon)/(\epsilon-1)}{k_o a\sqrt{\tau-1}\,(\tau\epsilon+\tau-\epsilon)}}{1+\dfrac{2\epsilon}{k_o a\sqrt{\tau-1}\,(\tau\epsilon+\tau-\epsilon)}}$$

c. Cutoff Wavelength (HE_{11})

$$\lambda_c = 2b\sqrt{\tau_c}$$

d. Proportionality Constant

$$\tau = \left(\frac{\lambda_o}{\lambda_g}\right)^2 + \left(\frac{\lambda_o}{2b}\right)^2$$

e. Characteristic equation

$$\tan\left(\frac{k_o a}{2}\sqrt{\epsilon-\tau}\right) = \frac{\epsilon\sqrt{\tau-1}}{\sqrt{\epsilon-\tau}}$$

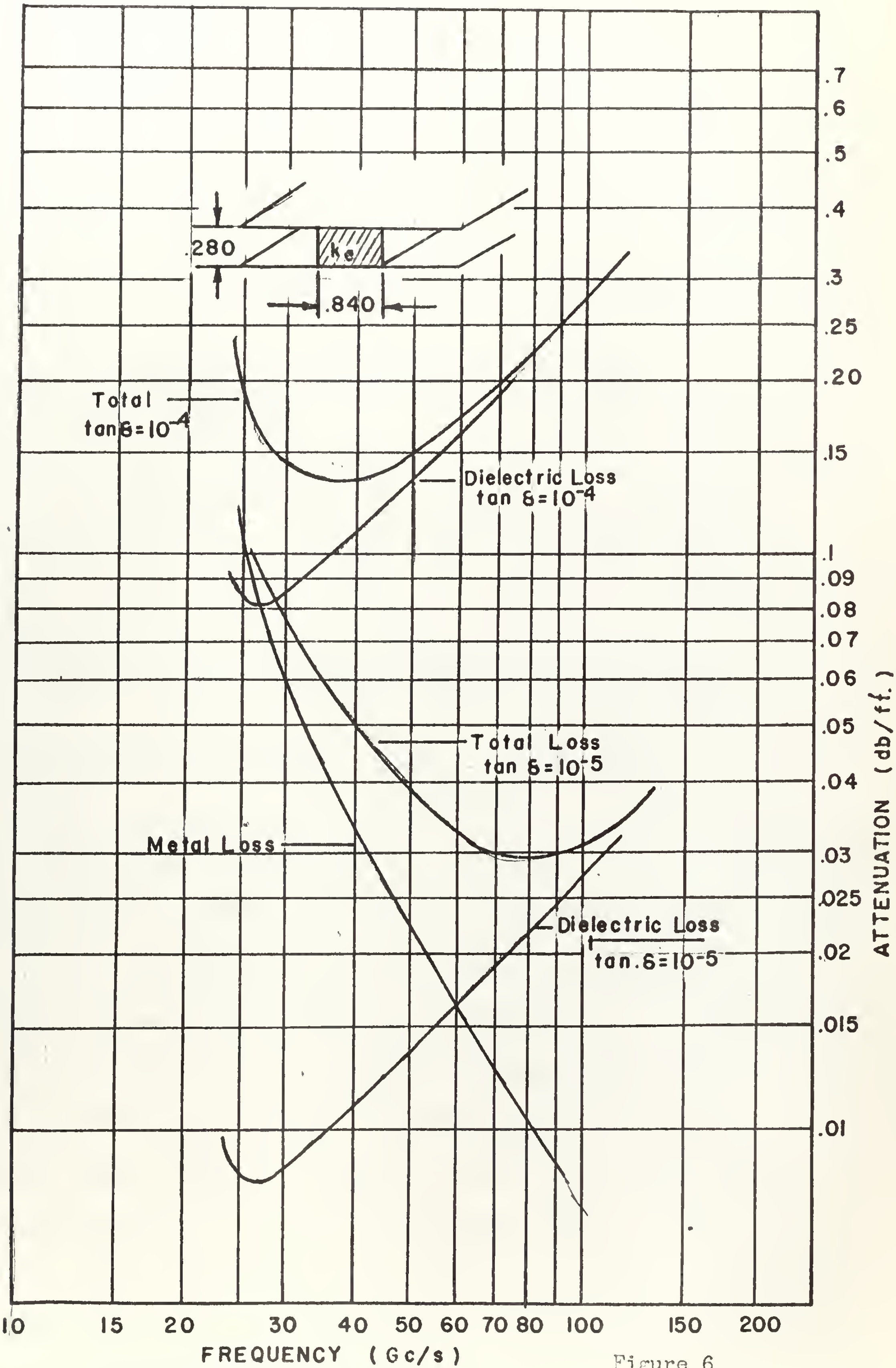


Figure 6

ATTENUATION OF H-GUIDE

2.6. Surface Wave Transmission Lines

The single wire transmission line has been known in theory since Sommerfeldt's work in 1899. It was realized by Goubau[29], but practical difficulties with bare wire have led to the development of a dielectric coated line[30,34] in which the dielectric is used to reduce the extent of the exponential field around the wire when the wire size is chosen for minimum attenuation. The conductor attenuation is:

$$\alpha_m = \frac{3.281 \times 10^{-5} P(k_c a')}{a\sqrt{\lambda}} \frac{db}{m} \qquad (2.6\text{-}1)$$

and the dielectric attenuation is:

$$\alpha_d = 3.281 \times 10^{-4} \left(\frac{1}{\frac{\epsilon_d'}{\epsilon_o} - 1}\right)\left(\frac{\sigma_d}{\omega \epsilon_d'}\right) \frac{\lambda}{a'^2} Q(k_c a') \quad db/m.$$

where

$$P(k_c a') = -1.33 \times 10^5 \left(\frac{\epsilon_o}{\mu_o}\right)^{1/2} \left(\frac{\pi \mu_c}{\sigma_c \mu_o}\right)^{\frac{1}{2}} \frac{1}{\ln k_c a' + .38}$$

$$Q(k_c a') = 2.11 \times 10^3 \left(1 - \frac{0.5}{\ln k_c a' + .38}\right)\left(k_c a'\right)^2$$

Excitation is normally accomplished by a horn much as is dielectric rod, creating an additional loss inversely proportional to frequency.

It can be seen that low loss is possible, though the line loss increases with frequency, bandwidth is adequate, but like dielectric line the surface wave line is subject to large radiation loss in bends except in the frequency sensitive cases of reflecting panels or critical radius bending. The guide is useful for nearly straight medium length runs; due to the accompanying fields it is not well suited to use in close spaces. In addition, the physical configuration of the line makes it more compatible with coaxial line than waveguide.

An interesting variation on the Goubau line is a coaxial type[43] in which a smaller wire does not touch the dielectric cylindrical sheath. This guide is more susceptable to physical nonuniformities than G-line, but has a wider band of low attenuation before the rise to all dielectric attenuation level commences. So far, constructional difficulties would seem to preclude the use of this line, since the original investigation was mathmatical only, and assumed air or free space between the wire and the dielectric which would be a difficult configuration to maintain, while the use of a solid dielectric would increase losses, particularly in the higher frequency region of use.

2.7. Multimode operation of Rectangular guide.

The section on circular electric mode operation has shown that overmoded hollow metal guide can be utilized to advantage under proper circumstances. Extending the use of rectangular guide beyond the normal upper frequency limit gives increasingly lower attenuation to the point at which wavelength is approximately .8a At $\lambda = a/2$, the theoretical attenuation is again about equal to the value at $\lambda = a$, the upper frequency limit assumed in paragraph 2.1. Recalling that rectangular guide attenuation varies inversely with guide height in the H_{10} mode suggests the possibility of using H_{01} mode to advantage. Figure 7 shows the H_{10} and H_{01} modes of RG-96 and the H_{01} mode of RG-52 to illustrate the possibilities of such operation.

If metal guide were not subject to mode conversion, only one restriction, other than the physical size of the guide, would limit the use of H_{01} mode to attain arbitrarily low attenuation; that limit is the point at which the attenuation of H_{01} becomes greater than that of H_{10} mode. This occurs at the cutoff frequency of H_{02}, the 14th mode

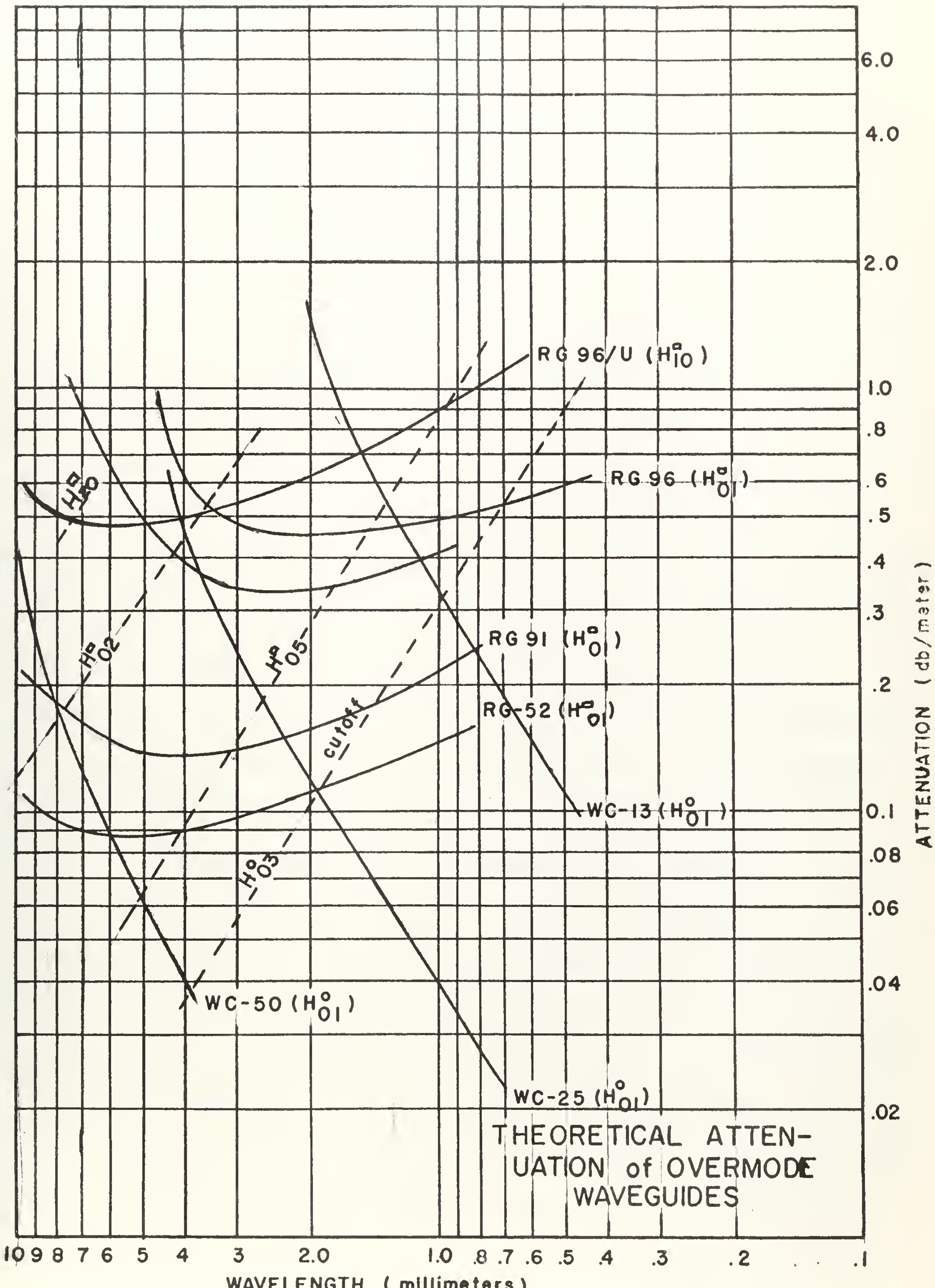
of rectangular guide of transverse proportions 2:1. In practice one cannot generally ignore the higher order modes as experience with circular guide has proved. In that guide the presently accepted practice for use in equipment design is to use guide limited to 25 propagating modes, since this allows propagation of only H_{01} and H_{02} through helix guide or filters, and the latter can be reasonably well filtered out or absorbed in the absence of higher order circular electric components. The propagating modes of rectangular guide can be computed*; indicative results are shown in figure 7. At the chosen limiting frequency and attenuation of circular guide, 375 modes can propagate, including the first ten H_{on} modes. Even though a process analogous to helix filtering could be used in rectangular guide, it is unlikely that economical operation could be maintained beyond the cutoff frequency of H_{05}, since by that point the theoretical attenuation of H_{01} is again increasing. Heller has reported the use of RG 96 at two millimeters[71] with no filtering other than the dominant mode guide used to feed the RG-96 through a smooth taper transition section. Although no quantitative measurements were made, no trouble was experienced with excessive losses due to mode conversion, and satisfactory reduction is attenuation was achieved.

Rectangular guide used in this manner could be expected to behave in a manner predictable by the same means as is used for circular electric mode and considered in Appendix A. Without specific mode conversion factors available, one can make only general statements of expectations, but these are as follows; bends should be of smaller

*. An empirical formula for the number of propagating modes in rectangular guide of a=2b proportions is:

$$M \doteq K(b/\lambda) \qquad C(b/\lambda)^2$$

where $K = -.67$ $\qquad C = 14.67$



RG 96/U ($H^{\square}_{10}$)
RG 96 ($H^{\square}_{01}$)
RG 91 ($H^{\square}_{01}$)
RG-52 ($H^{\square}_{01}$)
WC-13 (H^{o}_{01})
WC-50 (H^{o}_{01})
WC-25 (H^{o}_{01})
$H^{\square}_{20}$
$H^{\square}_{02}$
$H^{\square}_{05}$
H^{o}_{03}
cutoff
6.0
4.0
2.0
1.0
.8
.6
.5
.4
.3
.2
0.1
.08
.06
.04
.02
ATTENUATION (db/meter)
10 9 8 7 6 5 4 3 2.0 1.0 .8 .7 .6 .5 .4 .3 .2 .1
WAVELENGTH (millimeters)
THEORETICAL ATTENUATION of OVERMODE WAVEGUIDES

Figure 7

radius than comparable circular guide for the same loss in the bend, but such components as junctions would give rise to excessive higher order modes except over a narrow band, requiring the use of dominant mode or slightly oversize guide for these components. An interesting thought might be the use of H-guide at these points if the H-guide were of sufficiently low loss.

2.8. Quasi Optical transmission lines

The transmission lines enumerated in the preceding paragraphs give little hope of usefulness extending far into the submillimeter region, but reference to paragraph 3.0. shows that one other area is available and relatively untried. This is the use of planar, quasi-optical, or free-space techniques common to microwave antennas and optical equipment. It is normally assumed that an infinity of modes are transmitted in such a system, but in practice this is unimportant since all travel at the same speed. Assuming initially that any physical size requirements can be met, let us investigate this means of transmission.

Hertz demonstrated before the turn of the century that "electric waves" were subject to the same rules of reflection and refraction as visible light. More recently, microwave lenses and parabolic reflectors have been used. Although the lenses in current use at centimeter wavelength are of metal, they exhibit the properties of a dielectric to the passing wave. Kock of Bell Labs has made significant contributions in the field of these artificial dielectrics[24,67,68,69]. While such components may be less used in the submillimeter region where their strength and light weight are of less advantage, they illustrate the degree to which optical theory has been applied at longer wavelengths.

The transmission line here envisioned bears greatest resemblance

to a telescope; that is, the beam is directed by properly shaped and positioned lenses or mirrors with a surrounding structure whose primary purpose is to maintain the desired alignment of the active elements. Since metal tubing is necessarily highly reflecting, a lossy dielectric with a dielectric constant near unity would be preferred in this use. An advantage to this type of structure is its dimensional tolerance. While in waveguides we found the necessity for strict transverse conformity over the length of the guide, in a lens system, the critical dimensions are the longitudinal ones over the face of the active elements, Placement of these elements have tolerances more nearly on the order of focal length than of wavelength.

The performance of an optical system can be defined by the usual criteria of resolution, coma, and astigmatism for the purpose of determining necessary element size, but the small D/λ lenses to be used in the range near 1.0 mm. will allow quite large approximations to be made, leaving the limit of resolution the most significant of the three factors above. This is;

$$\delta_d = \frac{1.22\lambda}{D} \text{ radians} \qquad (2.8\text{-}1)$$

(or in an equivalent linear form)

$$d/\lambda = 1.22\, N, \text{ where } N = F/\text{number} = f/D \qquad (2.8\text{-}2)$$

For example, the minimum diameter to which a 1.0mm wave could be focussed with an F/1.0 lens is 1.22 mm. This limit in part accounts for the tendancy of microwave lenses to be of larger aperture than F/1.0.

Another property of lens systems important to use with radiating equipment is power handling capability; for a lens large enough to be effective, the radiated power can be considered to be spread over the

lens surface in some known distribution, the breakdown potential of the lens then being a function of the power per unit area at the peak of the power distribution curve. In general this approach leads one to the conclusion that the most likely point for breakdown is within the oscillator or feed system rather than the guide.

Although this system would be sufficient for many purposes, Goubau suggests that by the consideration of phase path length one can use a system of lenses for the transmission of coherent oscillations.[35] Experimental results reported by Goubau were based on lenses of $D/\lambda = 16$ and included a measured attenuation of .2 db/meter at 24 Gc/s, including losses in the launching and receiving horns which were not of optimum design. Calculated loss, not including the launching assemblies, was .05 db/meter. Extrapolating these figures into the one millimeter region, we find that direct scaling of all dimensions gives an attenuation proportional to f^{+1}, while changing frequency in guide of fixed dimensions results in attenuation proportional to $1/f$, it being assumed in both cases that dielectric constant remains sensibly constant over the frequencies in question, or that suitable substitution of lens material can be made to return the dielectric constant to its original value. The paper by Goubau supports the conclusion that lenses of 50 mm. diameter would provide sufficient aperture to meet the specifications that have been proposed in earlier sections for desired level of attenuation at wavelengths at and below one millimeter.

The bandwidth considerations in a lens system are based primarily on the choice of dielectric. If the index of refraction ,n, is defined; $n = \sqrt{\epsilon/\epsilon_0}$

where

$$\epsilon = \epsilon_0 + N\alpha \frac{f_0^2}{f_0^2 - f^2} \qquad (2.8\text{-}3)$$

it can be seen that in the range below $f = f_o/4$, the approximation $\epsilon = \epsilon_o + Na$ is valid, and there is no frequency dependance. Thus, to a good approximation, the frequency limits of the system for wideband use are those determined by loss of signal due to lack of resolution on the low frequency end, and the rapidly changing index of refraction as $f \to f_o/4$ on the high end. By proper choice of dielectric, this band can be made as wide as desired.

Phase correction to enable transmission of coherent oscillations is accomplished by locating phase correcting plates (lenses) in front of the source and before the image plane which delay the phase of the field by the amount

$$\varphi = -\frac{k}{2D} r^2 \qquad (2.8\text{-}4)$$

where r is the radial index, making the plates plano-convex lenses of maximum phase shift

$$\varphi_{max} = -\frac{\pi R^2}{\lambda D} \; ; \; Spacing = 2D \qquad (2.8\text{-}5)$$

In this form, D is the focal length of the phase plate, R is the radius. A normally computed optical lens is then located halfway between the phase plates to complete the system, which can be iterated as many times as necessary.

Changes in direction in optical systems are easily accomplished; changes through angles large enough to make efficient use of reflection can be made essentially lossless through use of plane mirrors. Angles of less than 90° are best accomplished by use of prisms, thus introducing losses by reflection and absorption similar in nature and degree to lens losses.

No special components have been developed for mating optical guides with coherent sources, but for essential uses such as mixer mounts, one could use a very short section of rectangular waveguide at the throat of the receiving horn. Specially treated mirrors, gratings, and other optical devices appear useful for such operations as polarizers, directional couplers, and attenuators.

3.0. Evaluation

Although the proceding section was divided into seven sections describing at least at many transmission lines, it can be seen that the lines described may be classified as one of two types, operating in one of three conditions, or as hybrid lines having components classifiable under more than one of the above headings. The guide types referred to are metallic and dielectric, the operating conditions are those described by the transverse dimension of the line, D, and may be called longitudinal ($D/\lambda \to 0$), transverse ($D/\lambda \doteq 1$), or planar ($D/\lambda \to \infty$). The unity of the transverse case is taken to be an extremely broad condition meaning a moderate finite number. Of the six basic cases, the longitudinal dielectric case is the least used, being exemplified by the VHF Goubau line. The other five classes are well known in at least one form each:

	metallic	dielectric
longitudinal	coaxial line	Goubau line at VHF (single wire surface wave)
transverse	waveguide	dielectric rod
planar	reflector	lens

There is little question of the unsuitability of metallic longitudinal line at UMW frequences, and surface wave single wire transmission lines at that frequency can no longer be classified as operating in the longitudinal fashion, so these classes will receive no further consideration.

3.1. Metallic waveguides

The transverse metallic lines have been the most suitable wave guides at microwave frequencies, and can be of use over the lower part of the UMW region if suitably chosen. Dominant mode use of RG-96

extends to 7.5 mm at an average attenuation of .5 db/meter. If we assume that this level of attenuation is tolerable in the choices to follow, the same guide can be used in H_{01} mode to one millimeter, although the mode restriction imposed in 2.7 would limit the operation to 1.5 mm, a more reasonable figure.

So long as knowledge of polarization is not required, the smaller number of modes in circular guide, and the greater present level of experience with it present some advantages over rectangular guide, but when these guides are equally well known the frequency and attenuation advantages of circular electric guide will be moderate. WC-13 should be about the smallest practicable guide among the standard sizes; in this case, the attenuation limit is imposed at the low frequency end of the desired 40% bandwidth of the guide, the upper frequency limit of the guide then being 1 mm. In this guide, there is an advantage to the perfection of wideband mode filters and absorbers to take further advantage of the decreasing attenuation as frequency increases. A more important need at present is the development of wideband components to operate in the guide. Current practice locates detectors, couplers, and other components in rectangular dominant mode guide, and restricts the use of circular guide to essentially straight connecting runs.

3.2. Dielectric lines

Dielectric rod guides have not encountered the mode conversion problems common to metallic guides primarily because low losses in the dielectric are dependent on the use of a cross-section smaller than a wavelength. Since the decay rate of the accompanying field is dependant on the cross-section in the opposite manner, the acceptable operating bandwidth of solid guide is extremely narrow. In all likelihood, solid

dielectric rod will find its greatest use as a laboratory patch cord as has been suggested at Bell Labs[20]. Although in this line guiding action may be small, some degree of polarization information can be transmitted by means of oval or rectangular cross-section line. The upper frequency limit of operation will depend on the physical requirements of the installation and their limiting effect on the minimum usable size of line.

The various hollow configurations of dielectric enable one to increase the size of the guide for some measure of rigidity, but their effect on bandwidth has been negligible. One attempt that has met success is the dielectric image line which takes advantage of the rigidity of the metal image plane to make use of foam dielectric tapes. By this means the bandwidth is increased and operation appears feasable to about 100 Gc/s. Manufacturing technique improvements can make further frequency increases possible; Wiltse has reported that the attenuation of a hand-made section of line was strongly influenced by the cement used to attach the dielectric to the image plane[64](in this case the thickness of the cement layer was nearly as great as the thickness of the dielectric).

3.3. Hybrid lines

H-guide is an excellent example of a hybrid under the classification system espoused in 3.0. The metal walls act in the same manner as do the walls of any other metallic transverse line, the dielectric slab is acted upon in a manner analogous to that of a dielectric rod in the same HE_{11} mode. The metal walls are parallel to the electric field in the same manner as the wall of circular electric mode, or the side walls of rectangular guide in H_{10} mode; that is, above a cutoff frequency

determined by the wall spacing the attenuation diminishes approximately as $f^{-3/2}$. The presence of the metal walls causes a definite cutoff frequency not found in other lines using the HE_{11} mode, but the dielectric can be used to restrain the fields and limit the number of propagating modes to a greater degree than could be done with the dielectric or metal alone. Fig. 8 shows some of the variations possible through altering slab dimensions and plate spacing. Additional changes could be achieved by the use of several smaller slabs, either of different dielectric constant or alternated with air. Present frequency outlook of this line is about 100Gc/s, but if dielectrics of tan $\delta = 10^{-5}$ become available, the attenuation will be reduced by a factor of 10 from that shown, allowing efficient use at higher frequencies. The ease of manufacture and joining, and the compatibility with other components could make this an interesting line, at that time to a frequency of about 300 Gc/s.

3.4. Summary of transitional methods

Figures 7 and 9, and table 3-1 summarize the transmission lines so far considered. In all cases it has been pointed out that operation in a particular mode is desirable, and in each case this restriction has led to a high frequency limitation in the low millimeter range even with the tacit asseumption of the ability to achieve in practice a useful line with an attenuation not much greater than the theoretical attenuations given, and in most cases the further assumption that standard components can be used or adapted for use with a particular guide. It is felt that these conditions can be met as manufacturers become better acquainted with the restricting factors in these components.

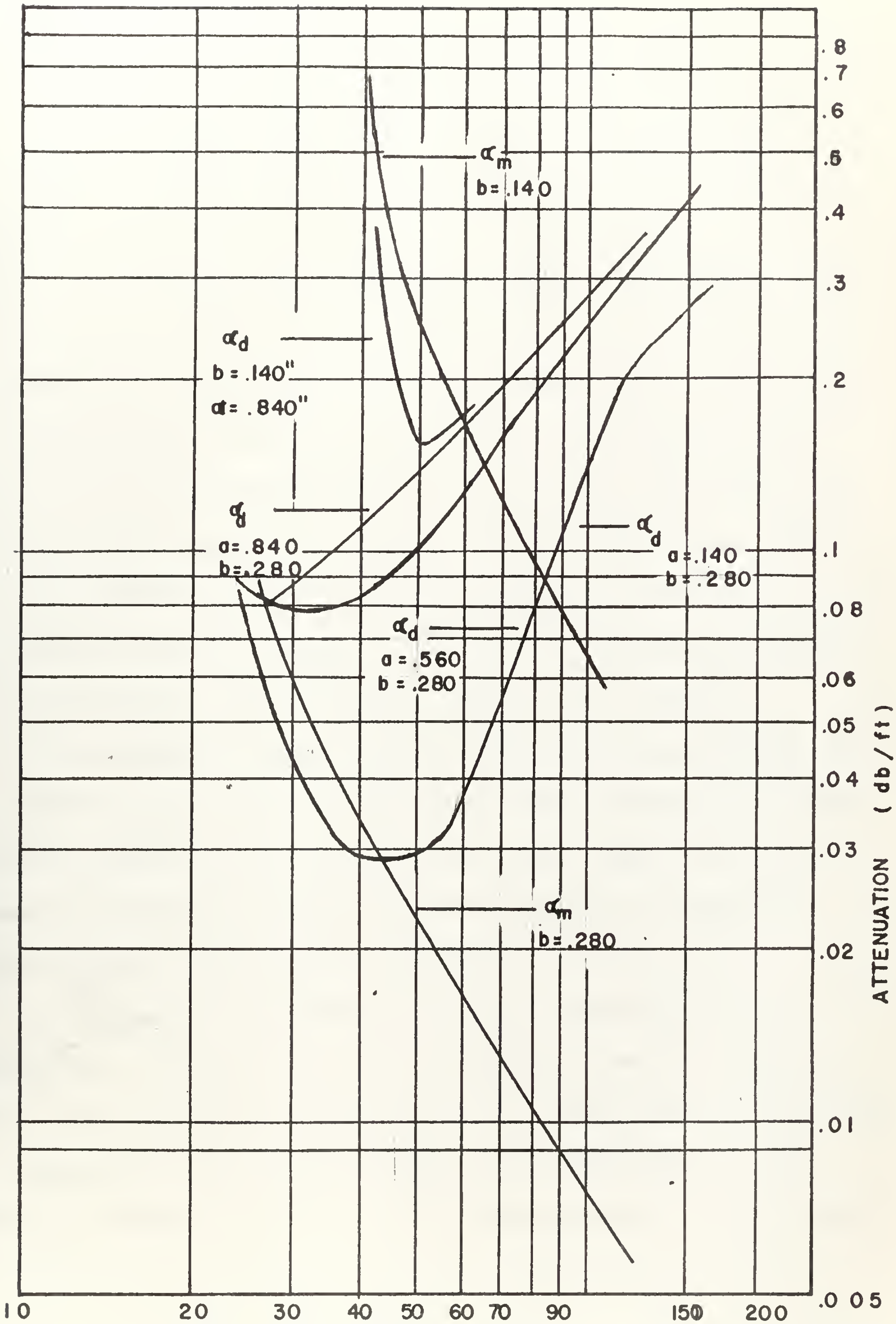


Figure 8

H-GUIDE ATTENUATION, VARYING SLAB SIZE

No attempt has been made to evaluate each possible mode or each variation on a transmission line; such a procedure is unnecessary, especially in the case of metallic waveguides. The lowest possible attenuation in such a guide is that encountered when the metal lies entirely parallel to the electric field lines of the mode. The corresponding attenuation varies as $f^{-3/2}$ in a given guide, and as $f^{+3/2}$ for a given operating condition (ie, $\lambda g/\lambda_o$ = a constant). Only one guide, the circular electric, has this lowest attenuation, but the rectangular H_{10} exhibits the effect to a marked degree by the decrease in attenuation with increasing guide height. As an example, RG-91 has a height only slightly greater than the width of RG-96, while the width of RG 91 is about four times the height of RG 96; fig. 7 shows that corresponding points on the H_{01} curve of RG 91 and the H_{10} curve of RG 96 are related by almost $f^{-3/2}$. The $f^{+3/2}$ dependence on guide size in rectangular guide is clearly shown in figs. 7 and 2.

Since attenuation must increase if the guide size is reduced, it is clear that only through the reduction of mode conversion losses can metallic waveguide be used beyond the millimeter range. The best of present techniques in manufacturing and laying will be required to attain 150 Gc/s with reasonable attenuation.

As indicated in the comments on H-guide, proper choice of dimensions in dielectric can achieve any desired level of attenuation over a band whose width is determined by the configuration of the guide. As frequency is increased: with $D/\lambda > 1$, the attenuation of a given strip of dielectric must vary as f^{+1}. With currently available foam dielectrics the resulting attenuation far from cutoff is comparable with that of dominant mode rectangular waveguide. Smaller attenuation values can be achieved only by use of thin dielectric tapes on an image

plane, by use of specially configured H-guide, or by use of lower loss dielectrics than are now available. A dielectric having a loss tangent of 10^{-5} would have an infinite medium ($D/\lambda > 1$) attenuation comparable to multimode waveguide, and would have the advantage of relative insensitivity to small changes in lateral dimensions. If used in H-guide the effect would be that shown in fig. 6 for the millimeter band; in dielectric image line such a dielectric could allow operation in the submillimeter band.

3.5. Conclusions

This paper has shown the millimeter band to be one of transition between the waveguide techniques of the centimeter band, and the optical techniques used at infrared and light frequencies. Either normal transmission line or optical line can be used, but some compromise must be made in each case. The dominant mode rectangular waveguide used at lower frequencies becomes excessively losst at about 40 Gc/s. A system of lenses with two inch diameter has too poor resolution for use below 200-300 Gc/s. Thus, the UMW region can be divided into three areas. Below 40 Gc/s, the equipment designer will continue to use the dominant mode rectangular guide primarily because its higher state of development outweighs any gains in attenuation possible through the use of other techniques. One possible exception to this choice exists in the case of long runs of guide in which the reduced attenuation of circular H_{01} guides could effect a worthwhile improvement in transmitted signal in spite of transitions and any filters necessary in this larger guide. As an alternative one could use H-guide or dielectric image line, but their higher susceptibility to degradation through the action of dust, moisture and spurious signals must be taken into account.

Between 40 and 300 Gc/s there exists no unique solution for an all purpose transmission line; the best line depends to large degree upon the specific use. The following observations are based on use in an equipment package where wide bandwidth is required. Overmoded rectangular guide can provide moderate losses to about 150 Gc/s while retaining polarization information and relatively low bending losses. Quantitative measurements and calculations are needed to properly evaluate the ultimate usefulness of this mode of operation. Circular guide gives low loss in straight sections to about 300 Gc/s, but polarization information is not transmitted, bending to radii less than about 15 diameters is normally detrimental to bandwidth, and the circular electric mode is not compatible with most receiving antennas. H-guide has all the properties of rectangular guide except complete shielding, and is capable of handling greater power. Present models are useful to 80 Gc/s, but refinement of constructional details could extend the range to at least 100 Gc/s. Development of less lossy dielectrics would allow operation past 200 Gc/s. For special purposes image line and dielectric rod could be useful in the same range. The most satisfactory transmission line within this region will still have greater attenuation than those of the ajoining bands, so the most satisfactory equipment designs will be those engineered to use as short a path at operating frequency as possible.

Once the operating frequency becomes high enough to allow the use of an optical system of reasonable diameter, about 200 Gc/s, it is likely that this system will prove an almost universally used line. Initially it will be used only as a two port line with horn transitions at each end, but as experience with it increases so should

Guide Type	Maximum Usable Frequency - a	90° bend loss	Band-width b	Remarks
Rectangular Dominant mode	40 Gc/s	1.06 SWR	40%	RG-96 taken as highest usable
Rectangular H_{01}	200 Gc/s	.2 db (est)	50%	a-f_c of H_{05} in RG 96 b-for RG 96. others can be larger, determined by allowable modes.
Circular H_{01}	300 Gc/s	.4 db	50%	b-for 2:1 change in attenuation. Minimum radius bend about 15 diameters for essentially pure mode output over this bandwidth.
H-Guide	100 Gc/s	$\doteq \alpha l_b$ (c)	50%	a. & b. depend on geometry of cross-section no optimum has been established. c. When bend plane includes both metallic sides. Planar bending of metal incurs higher loss length of linein bend.
Dielectric Image Line	100 Gc/s	$>2\alpha l_b$ (c)	note b	b - low pass filter action. Lowest frequency chosen by allowable field strength. c. $\alpha_b \doteq 2\alpha l_b$ in concave E bend; higher in all others
Optical line	lower limit 200-300 Gc/s note a	< 1.05 VSWR	note b	b. bandwidth dependant on dielectric used and lens shapes. Can be extremely wide. a. At 300 Gc/s, 2" dia lens assy. (not including feed loss)

TABLE 3-1. Summary of Transmission Line Characteristics for UMW.

the use of special components to accomplish the tasks of selective injection and coupling, detector mounting, and special purpose antennas. Guidelines in development of this equipment can come from infrared technique since the infrared region is commonly thought of as extending to the middle of the submillimeter band.

A great deal of effort can be profitably expended in investigating and improving transmission lines for the UMW region, but there is no part of the region that can not now be exploited due to the lack of a satisfactory transmission line.

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Appendix A

Mode coupling in Metal Wave guides

Some years ago Schelkunoff[70] described waveguide transmission in terms of "generalized telegraphists equations". At about the same time, Miller described the theory of mode coupling in waveguides.[54] The principles of these papers have been used to describe the mode coupling effects in circular waveguide[37], but could be used in the same manner to investigate the coupling in oversized rectangular guide. This section abstracts these papers sufficiently thoroughly to present the principles of coupling.

If we view a single waveguide as a system of transmission lines coupled to each other only by guide imperfections or discontinuities we find that the coupling can be expressed as the product of two quantities. One of these is a constant determined solely by the two modes under consideration, the other a proportionality factor describing the effectiveness of a specific nonuniformity in exciting the coupling. The first of these factors can be determined.

The space variation of amplitude in the modes may be written:

$$\begin{aligned} \frac{dE_1}{dx} &= -(\gamma_1 + k)E_1 + kE_2 \\ \frac{dE_2}{dx} &= kE_1 - (\gamma_2 + k)E_2 \end{aligned} \qquad \text{(A-1)}$$

where

$$\gamma_1 = \Gamma_1 + \Delta\Gamma_1$$

$$\gamma_2 = \Gamma_2 + \Delta\Gamma_2$$

and the Γ_i are the uncoupled propagation constants.

If all the energy is initially in mode 1 ; ie, $E_1 = 1.0$ $E_2 = 0$ at $x = 0$ we may write

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The space variation of amplitude in the modes may be written:

$$\frac{dE_1}{dz} = -(\gamma_1 + k)E_1 + kE_2$$

$$\frac{dE_2}{dz} = kE_1 - (\gamma_2 + k)E_2 \qquad (A-1)$$

where

$$\gamma_1 = \Gamma_1 + k_1'$$

$$\gamma_2 = \Gamma_2 + k_2$$

and the Γ_n are the uncoupled propagation constants.

If all the energy is initially in mode 1; ie, $E_1 = 1.0$ $E_2 = 0$ at $z = 0$ we may write

$$E_1 = \left[\frac{1}{2} - \frac{(\gamma_1 - \gamma_2)}{2\sqrt{(\gamma_1-\gamma_2)^2+4k^2}}\right]\epsilon^{r_1 x} + \left[\frac{1}{2} + \frac{(\gamma_1-\gamma_2)}{2\sqrt{(\gamma_1-\gamma_2)^2+4k^2}}\right]\epsilon^{r_2 x}$$

$$E_2 = \frac{k\ \epsilon^{r_1 x}}{\sqrt{(\gamma_1-\gamma_2)^2+4k^2}} - \frac{k\ \epsilon^{r_2 x}}{\sqrt{(\gamma_1-\gamma_2)^2+4k^2}} \qquad \text{(A-2)}$$

where

$$r_1 = -\tfrac{1}{2}(2k+\gamma_1+\gamma_2) + \tfrac{1}{2}\sqrt{(\gamma_1-\gamma_2)^2+4k^2}$$

$$r_2 = -\tfrac{1}{2}(2k+\gamma_1+\gamma_2) - \tfrac{1}{2}\sqrt{(\gamma_1-\gamma_2)^2+4k^2}$$

if it is then assumed that energy is conserved

$$|E_1|^2 + |E_2|^2 = K$$

k must be imaginary. An extention of this assumption leads to the convention that coupling always be considered non-dissipative, even though the modes themselves exhibit finite losses.

The equations above may be evaluated under a number of conditions on γ.

I. $\gamma_1 = \gamma_2$

$$E_1 = \cos cx\ \epsilon^{-(ic+\gamma)x} \qquad \text{(A-3)}$$

$$E_2 = i \sin cx\ \epsilon^{-(ic+\gamma)x} \qquad k = ic$$

Note that this is the condition of complete mode degeneracy and that the solution is a condition of complete cyclical transfer of energy. An example is the H^{o}_{01}---E^{o}_{11} degeneracy mentioned in the text.

II. $\alpha_1 = \alpha_2 = \alpha \qquad \gamma_1 - \gamma_2 = i(\beta_1 - \beta_2)$

$$E_1 = E_1'\ \epsilon^{-\left[\alpha + i\left(c + \frac{(\beta_1+\beta_2)}{2}\right)\right]x}$$

$$E_2 = E_2'\ \epsilon^{-\left[\alpha + i\left(c + \frac{\beta_1+\beta_2}{2}\right)\right]x}$$

$$E_1' = \cos\left[\sqrt{\frac{(\beta_1-\beta_2)^2}{4c^2}+1}\ cx\right] - \frac{i(\beta_1-\beta_2)}{2c}\cdot\frac{1}{\sqrt{\frac{(\beta_1-\beta_2)^2}{4c^2}+1}}\sin\left[\sqrt{\frac{(\beta_1-\beta_2)^2}{4c^2}+1}\ cx\right]$$

$$E_2' = \frac{1}{\sqrt{\frac{(\beta_1-\beta_2)^2}{4c^2}+1}}\sin\left[\sqrt{\frac{(\beta_1-\beta_2)^2}{4c^2}+1}\ cx\right]$$

In this case cyclical transfer is accomplished at a rate described by the period

$$\sqrt{\frac{(\beta_1-\beta_2)^2}{4c^2}+1}\; cx = \pi \qquad \text{(A-4)}$$

and the maximum power transferred decreases as $(\beta_1-\beta_2)$ differs increasingly from 0; for $|E_1'|^2 \geq .5\,|E_2'|^2$, $\dfrac{\beta_1-\beta_2}{c} \geq 2$ is required.

III $\quad \beta_1 = \beta_2 = \beta \qquad \gamma_1 - \gamma_2 = \alpha_1 - \alpha_2$

$$E_1 = \epsilon^{-[\alpha_1 + i(c+\beta)]x}\left\{\left[\frac{1}{2} - \frac{(\alpha_1-\alpha_2)}{2\sqrt{(\alpha_1-\alpha_2)^2-4c^2}}\right]\epsilon^{\left[\frac{(\alpha_1-\alpha_2)}{2}+\frac{1}{2}\sqrt{(\alpha_1-\alpha_2)^2-4c^2}\right]x}\right.$$

$$\left. + \left[\frac{1}{2} + \frac{(\alpha_1-\alpha_2)}{2\sqrt{(\alpha_1-\alpha_2)^2-4c^2}}\right]\epsilon^{\left[\frac{(\alpha_1-\alpha_2)}{2}-\frac{1}{2}\sqrt{(\alpha_1-\alpha_2)^2-4c^2}\right]x}\right\}$$

and

$$E_2 = \epsilon^{-[\alpha_1 + i(c+\beta)]x}\,\frac{ic}{\sqrt{(\alpha_1-\alpha_2)^2-4c^2}}\left\{\epsilon^{\left[\frac{(\alpha_1-\alpha_2)}{2}+\frac{1}{2}\sqrt{(\alpha_1-\alpha_2)^2-4c^2}\right]x}\right.$$

$$\left. - \epsilon^{\left[\frac{(\alpha_1-\alpha_2)}{2}-\frac{1}{2}\sqrt{(\alpha_1-\alpha_2)^2-4c^2}\right]x}\right\} \qquad \text{(A-5)}$$

There are two possible situations in this case; the more practical is $\alpha_1 < \alpha_2$. For this condition, maximum power transferred decreases as $\left|\dfrac{\alpha_1-\alpha_2}{c}\right|$ increases; when this factor is about 55, the maximum loss in the driven line is about $\frac{1}{4}$ db. In the case

of $\alpha_2 < \alpha_1$, driven mode loss can become quite irregular, the low attenuation mode periodically delivering and taking out considerable amounts of power from the driven mode.

IV. $\gamma_1 \neq \gamma_2$

$$E_1 = E_1^{***} \epsilon^{-[\alpha_1 + i(c + (\frac{\beta_1 + \beta_2}{2})]x}$$
$$E_2 = E_2^{***} \epsilon^{-[\alpha_1 + i(c + (\frac{\beta_1 + \beta_2}{2})]x} \tag{A-6}$$

This condition is best evaluated from the original equations, but the general action can be visualized as a combination of cases II and III.

Morgan's paper(37) is an excellent example of the use of this technique for determining the effects of mode conversion is curved circular waveguide having specific compensators installed. For the determination of expected losses in straight sections, one can extend the method after the manner used by Warters and Rowe(38).

One assumes that the coupling coefficients, c(x), have been computed for the modes considered; then

$$C_n = c_n(x) b_n, \quad \begin{array}{l} b_n = \text{deviation from straightness} \\ c_n = \text{mode coupling} \end{array}$$

for each discontinuity. Statistically, the average conversion loss can be expressed as

$$\langle \bar{A} \rangle = \frac{\hat{C}^2}{2} N, \quad \begin{array}{l} N = \text{number of points} \\ \hat{C}^2 = \text{rms value of } C_n = c(x)^2 \langle b^2 \rangle \end{array}$$

which can be considered a mean value about which the deviation to be expected is

$$\langle (\overline{\delta A})^2 \rangle = \frac{\hat{C}^4}{8} N(N-1)$$

Since the values above show discrete numbers of discontinuities, the continuous case is of interest in evaluating manufacturing tolerances.

[illegible]

Apply Fourier sampling such that

$$\Delta\beta(\lambda_k)L = 2\pi k$$

Then the loss, A_k, at the sample point, k, is

$$A_k = \frac{|C_k|^2}{2}L^2$$

and depends only on the k^{th} Fourier coefficient which in turn depends on the straightness of $2n/\Delta\beta$ sampling intervals. Finally, the mean and deviation are

$$\langle A_k\rangle = \frac{\langle |C_k|^2\rangle L^2}{2}$$

$$\langle(\delta A_k)\rangle = \frac{1}{2}\langle A_k\rangle^2 \qquad \text{(A-7)}$$